

SOLUTIONS

3

Choice Sets in Labor and Financial Markets

Solutions for *Microeconomics: An Intuitive Approach with Calculus*

Apart from end-of-chapter exercises provided in the student *Study Guide*, these solutions are provided for use by instructors. (End-of-Chapter exercises with solutions in the student *Study Guide* are so marked in the textbook.)

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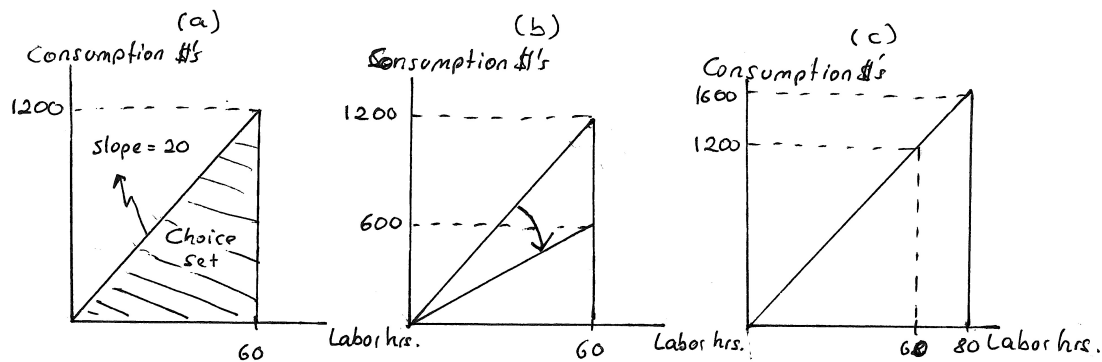
- Each end-of-chapter exercise begins on a new page. This is to facilitate maximum flexibility for instructors who may wish to share answers to some but not all exercises with their students.
- If you are assigning only the A-parts of exercises in *Microeconomics: An Intuitive Approach with Calculus*, you may wish to instead use the solution set created for the companion book *Microeconomics: An Intuitive Approach*.
- *Solutions to Within-Chapter Exercises are provided in the student Study Guide.*

3.1 In this chapter, we graphed budget constraints illustrating the trade-off between consumption and leisure.

A: Suppose that your wage is \$20 per hour and you have up to 60 hours per week that you could work.

- (a) Now, instead of putting leisure hours on the horizontal axis (as we did in Graph 3.1), put labor hours on the horizontal axis (with consumption in dollars still on the vertical). What would your choice set and budget constraint look like now?

Answer: Panel (a) of Graph 3.1 illustrates this choice set and budget constraint. It would begin at the origin (where no labor is provided and thus no income earned for consumption) and would rise by the wage rate (i.e. \$20) for each hour of labor.



Graph 3.1: Labor/Consumption Tradeoff

- (b) Where on your graph would the endowment point be?

Answer: The endowment point is always the bundle that a consumer can consume regardless of market prices (or wages). In this case, the bundle (0,0) — i.e. no labor and no consumption, is always possible. This is equivalent to the endowment bundle (60,0) when we put leisure instead of labor on the horizontal axis.

- (c) What is the interpretation of the slope of the budget constraint you just graphed?

Answer: The slope is equal to the wage rate (just as it is equal to the negative wage rate when leisure is graphed on the horizontal axis).

- (d) If wages fall to \$10 per hour, how does your graph change?

Answer: It changes as in panel (b) of Graph 3.1, with a new slope of 10 rather than 20.

- (e) If instead a new caffeine drink allows you to work up to 80 rather than 60 hours per week, how would your graph change?

Answer: Since wages have not changed, the graph would be identical up to 60 hours of work. But now 20 additional potential hours of work are possible — causing the budget constraint to extend all the way to 80 hours of labor. This is depicted in panel (c).

B: How would you write the choice set over consumption c and labor l as a function of the wage w and leisure endowment L ?

Answer: The choice set would be

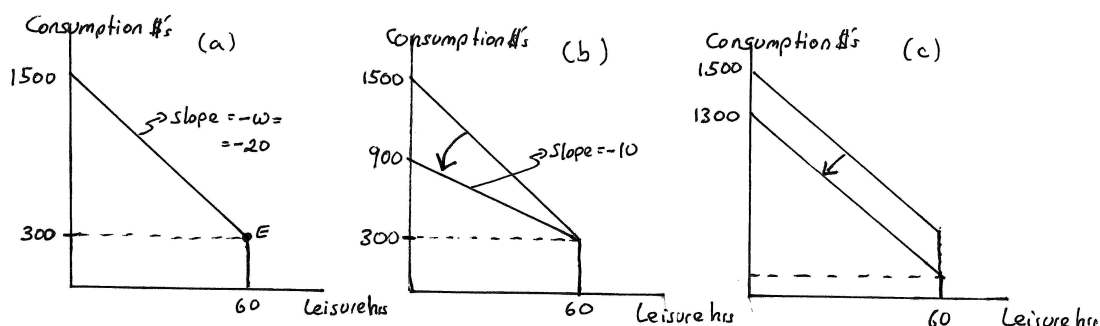
$$C(w, L) = \{(c, l) \in \mathbb{R}_+^2 \mid c \leq wl \text{ and } l \leq L\}. \quad (3.1)$$

3.2 In our treatment of leisure/consumption tradeoffs, we have assumed that you are deriving income solely from wages.

A: Suppose now that your grandparents set up a trustfund that pays you \$300 per week. In addition, you have up to 60 hours of leisure that you could devote to work at a wage of \$20 per hour.

(a) On a graph with "leisure hours per week" on the horizontal axis and "weekly consumption in dollars" on the vertical, illustrate your weekly budget constraint.

Answer: This is illustrated in panel (a) of Graph 3.2.



Graph 3.2: Leisure/Consumption Trade-off with Outside Income

(b) Where in your graph is your endowment bundle?

Answer: The endowment bundle is always available regardless of market prices. This bundle is (60,300), labeled E in panel (a) of the graph.

(c) How does your graph change when your wage falls to \$10?

Answer: This is illustrated in panel (b) of Graph 3.2.

(d) How does the graph change if instead the trust fund gets raided by your parents, leaving you with only a \$100 payment per week?

Answer: This is illustrated in panel (c) of Graph 3.2.

B: How would you write your budget constraint described in 3.2A?

Answer: The constraint is written as

$$c = 1500 - 20\ell \text{ and } \ell \leq 60.$$

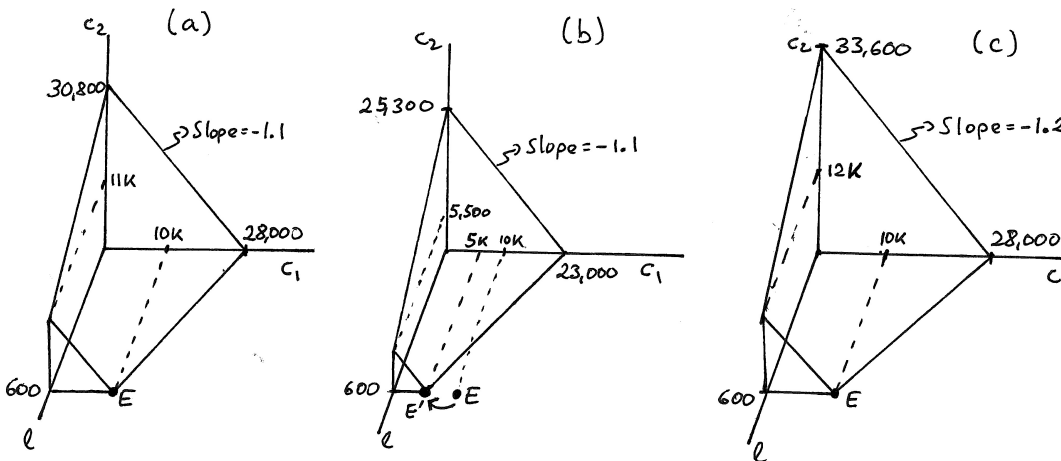
(3.2)

3.3 You have \$10,000 sitting in a savings account, 600 hours of leisure time this summer and an opportunity to work at a \$30 hourly wage.

A: Next summer is the last summer before you start working for a living, and so you plan to take the whole summer off and relax. You need to decide how much to work this summer and how much to spend on consumption this summer and next summer. Any investments you make for the year will yield a 10% rate of return over the coming year.

- (a) On a three dimensional graph with this summer's leisure (ℓ), this summer's consumption (c_1) and next summer's consumption (c_2) on the axes, illustrate your endowment point as well as your budget constraint. Carefully label your graph and indicate where the endowment point is.

Answer: This is graphed in panel (a) of Graph 3.3. The endowment point — the point on the budget constraint that is always available regardless of prices — is $(\ell, c_1, c_2) = (600, 10000, 0)$ where no work is done and all \$10,000 in the savings account is consumed now. No matter what the wage or the interest rate is, you can always consume this bundle. However, you can also work up to 600 hours this summer, which would earn you up to an additional \$18,000 for consumption now. So the most you could consume this summer if you worked all the time and emptied your savings account is \$28,000, the c_1 intercept. You can't consume any more than 600 hours of leisure — so 600 is the ℓ intercept. If you earned \$18,000 this summer (by working all the time and thus consuming zero leisure), and if you consumed nothing this summer, then the most you could consume next summer is \$28,000 times $(1+r)$ where $r = 0.1$ is the interest rate. This gives the c_2 intercept of 30,800. If you don't work (i.e. $\ell = 600$) and you consume nothing this summer (i.e. $c_1 = 0$), then you simply have your \$10,000 from your savings account plus \$1,000 in interest next summer, for a total of \$11,000 in c_2 . The overall shape of the budget constraint then becomes the usual triangular shape but, because you can't buy leisure beyond 600 hours with your savings, the tip of the triangle is cut off.



Graph 3.3: Leisure/Consumption and Intertemporal Tradeoffs Combined

- (b) How does your answer change if you suddenly realize you still need to pay \$5,000 in tuition for next year, payable immediately?

Answer: This is graphed in panel (b). Since \$5,000 has to leave your savings account right now, this leaves you with only \$5,000 in the account and thus shifts your endowment point in. The rest of the budget constraint is derived through similar logic to what was used above.

The budget constraint is again a triangular plane with the tip cut off, only now the tip that's cut off is smaller. (Had the immediately-due tuition payment been \$10,000, the cut-off tip would have disappeared.)

(c) *How does your answer change if instead the interest rate doubles to 20%?*

Answer: This is illustrated in panel (c) of Graph 3.3 where E is back to what it was in part (a) but the amount of consumption next summer goes up because of the higher interest rate. The reasoning for the various intercepts is similar to that above.

(d) *In (b) and (c), which slopes are different than in (a)?*

Answer: Slopes are formed by ratios of prices — so the only way that slopes can change is if a price has changed. In the scenario in (b), no price has changed. Thus, the only thing that happens is that E shifts in as indicated in the graph. This changes the various intercepts, but the slopes in each plane are parallel to those from panel (a). In the scenario in (c), on the other hand, the interest rate changes. The interest rate is a price that is reflected in any intertemporal budget constraint — i.e. any budget constraint that spans across time periods. In our graph in panel (c), this includes the constraint that lies in the plane that shows the tradeoff between c_1 and c_2 , and the plane that shows the tradeoff between ℓ (which is leisure *this* summer) and c_2 . Those slopes change as the interest rate changes, but the slope in the plane that illustrates the tradeoff between ℓ and c_1 does not — because that tradeoff happens within the same time period and thus does not involve interest payments.

B: *Derive the mathematical expression for your budget constraint in 3.3A and explain how elements of this expression relate to the slopes and intercepts you graphed.*

Answer: An intuitive way to construct this mathematical expression involves thinking about how much consumption c_2 is possible next summer. If you consume ℓ amount of leisure this summer, you will have a total of $10000 + 30(600 - \ell)$ available for consumption *this* summer — the \$10,000 in the savings account plus your earnings (at a wage of \$30) from hours that you did not consume as leisure. When we take this amount and subtract from it the consumption c_1 you actually undertake this summer, we get the amount that will be in the savings account for the year to accumulate interest. Thus, next summer, you will have $10000 + 30(600 - \ell) - c_1$ times $(1 + r)$ (where $r = 0.1$ is the interest rate); i.e.

$$c_2 = 1.1 [10000 + (30)(600 - \ell) - c_1] = 30800 - 33\ell - 1.1c_1, \quad (3.3)$$

or, written differently,

$$1.1c_1 + c_2 + 33\ell = 30800. \quad (3.4)$$

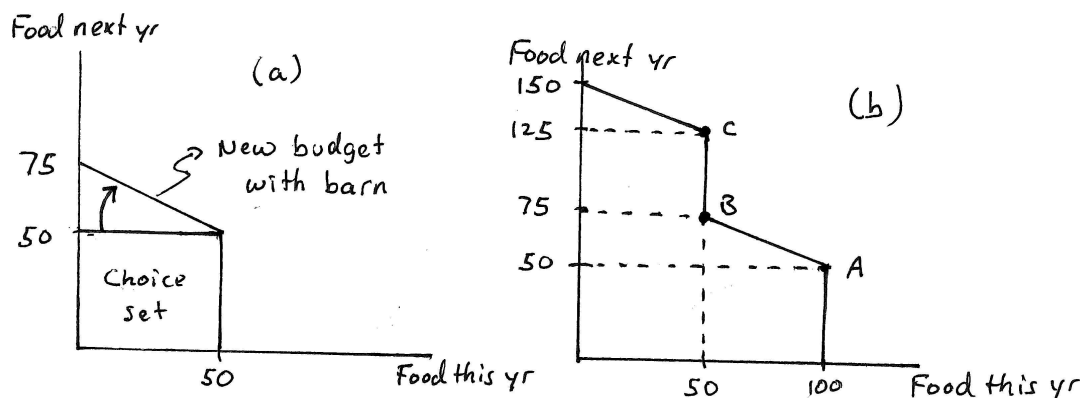
The only caveat to this when we define the budget plane is that we have to take into account that you cannot consume more than 600 hours of leisure (or negative quantities of consumption). We can incorporate this by defining the budget plane as

$$\{(c_1, c_2, \ell) \in \mathbb{R}_+^3 \mid 1.1c_1 + c_2 + 33\ell = 30800 \text{ and } \ell \leq 600\}. \quad (3.5)$$

3.4 Suppose you are a farmer whose land produces 50 units of food this year and is expected to produce another 50 units of food next year. (Assume that there is no one else in the world to trade with.)

A: On a graph with “food consumption this year” on the horizontal axis and “food consumption next year” on the vertical, indicate your choice set assuming there is no way for you to store food that you harvest this year for future consumption.

Answer: The initial choice set is simple because there is no way to trade off consumption between now and the future. Thus, the choice set involves all consumption levels with less than 50 units of food each year — equivalent to the square box labeled “Choice Set” in panel (a) of Graph 3.4.



Graph 3.4: Farming, Barns and Cows without Refrigeration

(a) Now suppose that you have a barn in which you can store food. However, over the course of a year, half the food that you store spoils. How does this change your choice set?

Answer: With the barn, you could now increase consumption up to 75 units of food next year if you did not eat this year and put all your 50 units of current food into the barn — with half of that surviving to next year. But since you can't store next year's food backward in time, you still can't consume more than 50 units of food now. The change in the choice set because of the barn is also depicted in panel (a) of the graph.

(b) Now suppose that, in addition to the food units you harvest off your land, you also own a cow. You could slaughter the cow this year and eat it for 50 units of food. Or you could let it graze for another year and let it grow fatter — then slaughter it next year for 75 units of food. But you don't have any means of refrigeration and so you cannot store meat over time. How does this alter your budget constraint (assuming you still have the barn from part (a))?

Answer: This is a bit more complex. We can begin by asking what is the most food you could consume this year — which would be 100 units, the 50 from farming and another 50 from slaughtering the cow. If you chose to consume all of that this year, you would just have the 50 units of food from farming next year — leaving you with bundle A in panel (b) of Graph 3.4. Alternatively, you could slaughter the cow — giving you 50 units of food — and store some of the farmed food in your barn. This produces the consumption possibilities between A and B in the graph, with decreases in current consumption fully coming from you putting farmed food into the barn and getting half of that (the half that does not spoil) back out of the barn next year. At B, you are just consuming the cow (50 units) and putting all your farmed food into the barn — giving you 25 units of food from the barn next year, in addition to the 50 units you'll farm next year, for a total of 75 units next year. Or you could save the cow and let it grow fat but consume all of your farmed food this year. That would give you 50 units of (farmed) food this year and 125 units of food next year — 50 from food

you'll farm next year plus 75 from the fattened cow. That's bundle C in the graph. Finally, you could put additional food in the barn (thus reducing your food consumption this year below 50) and get back half of whatever you put in the barn next year (in addition to the fattened cow and the amount you'll farm next year.) If you decide not to eat this year, the most you can get next year is the 50 you'll farm next year, the 75 from the fattened cow and an additional 25 from the unspoiled portion of this year's farmed food that you put in the barn — for a total of 150 units of food next year.

B: How would you write the choice set you derived in A(b) above mathematically, with c_1 indicating this year's food consumption and c_2 next year's food consumption?

Answer: This could be written as

$$\{ (c_1, c_2) \in \mathbb{R}_+^2 \mid \begin{array}{ll} c_1 \leq 100 & \text{and} \\ c_2 \leq 150 - 0.5c_1 & \text{for } c_1 \leq 50 \text{ and} \\ c_2 \leq 100 - 0.5c_1 & \text{for } 50 < c_1 \leq 100 \end{array} \}. \quad (3.6)$$

3.5 Suppose you are a carefree 20-year old bachelor whose lifestyle is supported by expected payments from a trust fund established by a relative who has since passed away. The trust fund will pay you \$ x when you turn 21 (a year from now), another \$ y when you turn 25 and \$ z when you turn 30. You plan to marry a rich heiress on your 30th birthday and therefore only have to support yourself for the next 10 years. The bank that maintains the trust account is willing to lend money to you at a 10% interest rate and pays 10% interest on savings. (Assume annual compounding.)

A: Suppose $x = y = z = 100,000$.

(a) What is the most that you could consume this year?

Answer: For the initial \$100,000 you get next year when you turn 21, the bank would be willing to lend you $\$100,000/(1+0.1) = \$90,909.09$. For the \$100,000 you get 5 years from now when you turn 25, the bank would be willing to lend you $\$100,000/(1+0.1)^5 = \$62,092.13$. And the bank would lend you up to $\$100,000/(1+0.1)^{10} = \$38,554.33$ for the \$100,000 you get on your 30th birthday (10 years from now). That sums to \$191,555.55.

(b) What is the most you could spend at your bachelor party 10 years from now if you find a way to live without eating?

Answer: If you saved your initial \$100,000 for 9 years (until your 30th birthday around which you will have your bachelor's party), you would have accumulated $\$100,000(1+0.1)^9 = \$235,794.77$. If you save the \$100,000 you get on your 25th birthday for 5 years, you would accumulate $\$100,000(1+0.1)^5 = \$161,051$. Add those two amounts to the \$100,000 you get when you are 30, and you get that you could spend a total of \$496,845.77 on your bachelor's party.

B: Define your 10-year intertemporal budget constraint mathematically in terms of x , y and z , letting c_1 denote this year's consumption, c_2 next year's consumption, etc. Let the annual interest rate be denoted by r .

Answer: You can think of this in the following way: First, how much will you have in wealth on your 30th birthday if you spend nothing and save everything? You will have had x in the bank for 9 years and y for 5 years — in addition to just having received z (on your 30th birthday). Thus, you will have

$$\text{Potential Wealth on 30th Birthday} = (1+r)^9 x + (1+r)^5 y + z. \quad (3.7)$$

Next, we can ask how much would be available for consumption in the year that starts with your 29th birthday (assuming you have to borrow whatever you intend to spend at the beginning of that year). Since we know how much wealth you would have on your 30th birthday if you did not spend anything leading up to it, we know that the most you can consume in the year before is how much you could borrow on that wealth; which means that the most you could consume in year 10, c_{10}^{max} , is an amount that would allow you to pay back $(1+r)^9 x + (1+r)^5 y + z$ one year from then — i.e.

$$(1+r)c_{10}^{max} = (1+r)^9 x + (1+r)^5 y + z. \quad (3.8)$$

The most you can consume in year 9, c_9^{max} , is similarly what you could pay back on your 30th birthday (with 2 years of interest) minus what you actually consumed in year 10 (c_{10}) (plus one year interest) — i.e.

$$(1+r)^2 c_9^{max} + (1+r)c_{10} = (1+r)^9 x + (1+r)^5 y + z. \quad (3.9)$$

Continuing this same logic backwards, we then get the 10-year intertemporal budget constraint

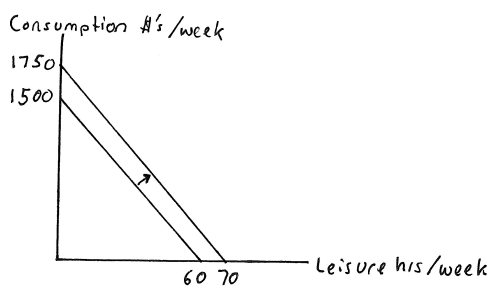
$$(1+r)^{10} c_1 + (1+r)^9 c_2 + \dots + (1+r)^2 c_9 + (1+r)c_{10} \leq (1+r)^9 x + (1+r)^5 y + z. \quad (3.10)$$

3.6 Everyday Application: Robots as Labor Saving Products: Suppose that you have 60 hours per week of leisure time and that you can earn \$25 per hour in the labor market. Part of the reason you do not have more time to work is that you need to do a variety of household chores: cleaning, shopping for food, cooking, laundry, running errands, etc. Suppose that those chores take 20 hours of your time per week. Suddenly you see an advertisement in the newspaper: “Personal Robot can do the following: clean, shop, cook, do laundry, run errands, etc. Can be rented by the week.”

A: Suppose you learn that the weekly rental fee is \$250 and that the robot could indeed do all the things that you currently spend 20 hours per week doing (outside the 60 hours of leisure you could be taking).

- (a) Illustrate your new weekly budget constraint assuming you decide to rent the robot. Be sure to incorporate the fact that you have to pay \$250 each week for the robot but assume that there is no consumption value in having a robot other than the time you are saved doing chores you would otherwise have to be doing. Are you better off with or without the robot?

Answer: Graph 3.5 illustrates the original as well as the new budget constraint. The robot would cost \$250, or 10 hours worth of work. At the same time, the robot adds 20 hours of leisure. Thus, after paying for the rental of the robot, you are left with 10 additional hours of leisure. Since your entire old choice set is fully contained in the new budget set, the robot must be making you better off.



Graph 3.5: Robots as Labor Saving Devices

- (b) As it turns out, everyone else wants this robot as well — and so the rental price has increased to \$500 per week. How does this change your answer?

Answer: In that case, it would take 20 hours to earn enough to pay for the robot rental. Thus, while the robot would increase leisure endowment by 20, the time it takes to pay for the robot would decrease it by 20. This would leave us with the original leisure endowment of 60 and the original budget constraint in the graph.

B: Incorporate the impact of the robot into the budget equation and illustrate how it leads to the graph you derived in 3.6A(a).

Answer: Without the robot, the budget constraint is $c = 25(60 - \ell) = 1500 - 25\ell$. When we can rent the robot for \$250 per week, leisure goes up to 80 but the rental fee has to be paid. This turns the budget constraint to $c = 25(80 - \ell) - 250 = 1750 - 25\ell$. Thus, the c intercept increases from 1,500 to 1,750 while the slope remains unchanged.

3.7 Everyday Application: Investing for Retirement: Suppose you were just told that you will receive an end-of-the-year bonus of \$15,000 from your company. Suppose further that your marginal income tax rate is 33.33% — which means that you will have to pay \$5,000 in income tax on this bonus. And suppose that you expect the average rate of return on an investment account you have set up with your broker to be 10% annually (and, for purposes of this example, assume interest compounds annually.)

A: Suppose you have decided to save all of this bonus for retirement 30 years from now.

- (a) In a regular investment account, you will have to pay taxes on the interest you earn each year. Thus, even though you earn 10%, you have to pay a third in taxes — leaving you with an after-tax return of 6.67%. Under these circumstances, how much will you have accumulated in your account 30 years from now?

Answer: You will have $\$10000(1 + 0.06667)^{30} = \$69,327$. Since you have already paid taxes on the initial bonus and on all interest income, no further taxes are due — so all \$69,327 is available for consumption.

- (b) An alternative investment strategy is to place your bonus into a 401K “tax-advantaged” retirement account. The federal government has set these up to encourage greater savings for retirement. They work as follows: you do not have to pay taxes on any income that you put directly into such an account if you put it there as soon as you earn it, and you do not have to pay taxes on any interest you earn. Thus, you can put the full \$15,000 bonus into the 401K account, and you can earn the full 10% return each year for the next 30 years. You do, however, have to pay taxes on any amount that you choose to withdraw after you retire. Suppose you plan to withdraw the entire accumulated balance as soon as you retire 30 years from now, and suppose that you expect you will still be paying 33.33% taxes at that time. How much will you have accumulated in your 401K account, and how much will you have after you pay taxes? Compare this to your answer to (a) — i.e. to the amount you would have at retirement if you saved outside the 401K plan.

Answer: Your account will grow to $\$15000(1 + 0.1)^{30} = \$261,741$. But you still have to pay one third in taxes — leaving you with $(2/3) * (\$261,741) \approx \$174,503$. This is substantially larger than the amount of \$69,327 we calculated in part (a).

- (c) True or False: By allowing individuals to defer paying taxes into the future, 401K accounts result in a higher rate of return for retirement savings.

Answer: This is true. In both cases, you end up paying taxes on all your income — both the initial income as well as interest income. The only difference between the two investment strategies is that in one case income is taxed as it is made, in the other case it is taxed at the end when it is withdrawn for consumption. In the latter case, the investor benefits from accumulating more interest faster. In our example, for instance, you end up with \$105,175 more with a 401K account than in a non-tax advantaged account.

B: Suppose more generally that you earn an amount I now, that you face (and will face in the future) a marginal tax rate of t (expressed as a fraction between 0 and 1), that the interest rate now (and in the future) is r and that you plan to invest for n periods into the future.

- (a) How much consumption will you be able to undertake n years from now if you first pay your income tax on the amount I , then place the remainder in a savings account whose interest income is taxed each year. (Assume you add nothing further to the savings account between now and n years from now).

Answer: You would place $(1 - t)I$ into the account, and it would earn an after-tax rate of return of $(1 - t)r$. Over n years, this results in $(1 - t)I(1 + (1 - t)r)^n$.

- (b) Now suppose you put the entire amount I into a tax-advantaged retirement account in which interest income can accumulate tax-free. Any amount that is taken out of the account is then taxed as regular income. Assume you plan to take the entire balance in the account out n years from now (but nothing before then). How much consumption can you fund from this source n years from now?

Answer: You would then accumulate $I(1 + r)^n$, but that amount would then be taxed at the end. So, what you are left with would be $(1 - t)I(1 + r)^n$.

- (c) Compare your answers to (a) and (b) and indicate whether you can tell which will be higher.

Answer: In both cases, a quantity $(1 - t)I$ is multiplied by another term in parentheses. In the case of no tax-advantaged treatment, this second term is $(1 + (1 - t)r)^n$; in the 401K case,

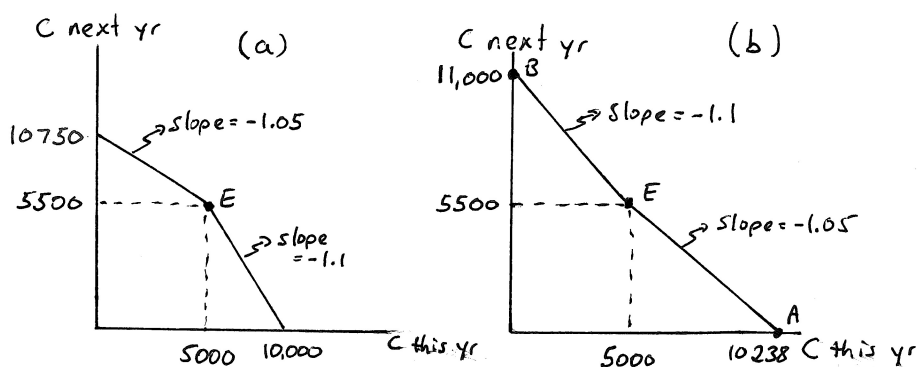
it is $(1 + r)^n$. So the latter is bigger so long as $(1 + r)$ is larger than $(1 + (1 - t)r)$ which is equal to $(1 + r - tr)$. When expressed this way, it is clear that the latter is smaller by tr — tax advantaged savings accounts always result in larger future consumption for given levels of investment.

3.8 Everyday Application: Different Interest Rates for Borrowing and Lending. Suppose we return to the example from the text in which you earn \$5,000 this summer and expect to earn \$5,500 next summer.

A: In the real world, banks usually charge higher interest rates for borrowing than they will give on savings. So, instead of assuming that you can borrow and lend at the same interest rate, suppose the bank pays you an interest rate of 5% on anything you save but will lend you money only at an interest rate of 10%. (In this exercise, it helps to not draw everything to scale much as we did not draw intertemporal budgets to scale in the chapter.)

- (a) Illustrate your budget constraint with consumption this summer on the horizontal and consumption next summer on the vertical axis.

Answer: Panel (a) of Graph 3.6 illustrates this intertemporal budget constraint. If you save all \$5,000 of this summer's income, you will have $\$5,000(1.05) = \$5,250$ in your savings account next year. Added to your \$5,500 income from next summer, that would let you spend a total of \$10,750 — the intercept on the vertical axis. If, on the other hand, you borrowed on your entire income from next summer, the bank would lend you up to $\$5,500/1.1 = \$5,000$. Added to your current income of \$5,000, that would give you a total of \$10,000 to spend this summer — the intercept on the horizontal axis. The slopes of the budget constraint change at E because that is the point at which you change from a saver to a borrower.



Graph 3.6: Different interest rates for borrowing and lending

- (b) How would your answer change if the interest rates for borrowing and lending were reversed?

Answer: This is illustrated in panel (b) of Graph 3.6. The reasoning for the values on the intercept is similar to that for part (a) except that now the role of the interest rates are reversed.

- (c) A set is defined as "convex" if the line connecting any two points in the set also lies in the set. Is the choice set in part (a) a convex set? What about the choice set in part (b)?

The set in part (a) is a convex set. The set in part (b) is not a convex set because I can find points contained in the set such that the line connecting those points does not lie in the set. Take, for instance, the bundles A and B — the intercepts of the budget constraint. Because the kink at E points inward, the line connecting A and B lies outside the choice set.

- (d) Which of the two scenarios would you prefer? Give both an intuitive answer that does not refer to your graphs and demonstrate how the graphs give the same answer.

Answer: You would prefer scenario (b) to scenario (a). It is easy to see in the graphs that this must be the case, particularly when panels (a) and (b) are placed on top of one another. The entire choice set in (a) is fully contained in the choice set in (b) — but the choice set in (b) has additional bundles that are possible. Any expansion of the choice set should be

at least as good and probably better. Intuitively, it should make sense that we would prefer (b). The interest rate on savings is an interest rate we in essence charge the bank while the interest rate on borrowing is an interest rate the bank charges us. We would always prefer prices that we charge others to be high and prices others charge to us to be low. In scenario (b), the price charged by us (the interest rate on savings) is high while the price charged to us (the interest rate on borrowing) is low — the best of all worlds.

B: Suppose more generally that you earn e_1 this year and e_2 next year and that the interest rate for borrowing is r_B and the interest rate for saving is r_S . Let c_1 and c_2 denote consumption this year and next year.

(a) Derive the general expression for your intertemporal choice set under these conditions.

Answer: If $e_1 > c_1$, you are a saving the difference; if $e_1 < c_1$, you are borrowing the difference. We can use this insight to write the choice set generally as

$$C(e_1, e_2, r_S, r_B) = \{(c_1, c_2) \in \mathbb{R}_+^2 \mid \begin{array}{ll} c_2 = (1 + r_S)(e_1 - c_1) + e_2 & \text{for } e_1 \geq c_1 \text{ and} \\ c_2 = (1 + r_B)(e_1 - c_1) + e_2 & \text{for } e_1 \leq c_1 \end{array} \}. \quad (3.11)$$

(b) Check that your general expression is correct by substituting the values from A(a) and (b) and checking that you get a choice set similar to those you derived intuitively.

Answer: In part A, $e_1 = 5000$ and $e_2 = 5500$. Initially, in A(a), we then consider the case where $r_B = 0.1$ and $r_S = 0.05$. Plugging these into the expression above, we get

$$C(5000, 5500, 0.05, 0.10) = \{(c_1, c_2) \in \mathbb{R}_+^2 \mid \begin{array}{ll} c_2 = (1.05)(5000 - c_1) + 5500 & \text{for } e_1 \geq c_1 \text{ and} \\ c_2 = (1.10)(5000 - c_1) + 5500 & \text{for } e_1 \leq c_1 \end{array} \}. \quad (3.12)$$

Take the first equation in the expression — which refers to cases where $c_1 \leq 5000$ — i.e. when you are a saver and thus consume less than \$5,000 this summer. That equation is

$$c_2 = (1.05)(5000 - c_1) + 5500 = 5250 - 1.05c_1 + 5500 = 10750 - 1.05c_1, \quad (3.13)$$

which is precisely what is graphed for $c_1 \leq 5000$. The second equation in the expression refers to $c_1 \geq 5000$ and becomes

$$c_2 = (1.10)(5000 - c_1) + 5500 = 5500 - 1.1c_1 + 5500 = 11000 - 1.1c_1. \quad (3.14)$$

If you extended the line to the left of E in panel (a) of Graph 3.6 to the vertical axis, you would indeed find a vertical intercept of 11,000 with the slope of -1.1 leading to a horizontal intercept of 10,000.

In A(b), we consider the case where $r_B = 0.05$ and $r_S = 0.10$. Plugging these into the expression from our solution to part B(a) above, we get

$$C(5000, 5500, 0.10, 0.05) = \{(c_1, c_2) \in \mathbb{R}_+^2 \mid \begin{array}{ll} c_2 = (1.10)(5000 - c_1) + 5500 & \text{for } e_1 \geq c_1 \text{ and} \\ c_2 = (1.05)(5000 - c_1) + 5500 & \text{for } e_1 \leq c_1 \end{array} \}. \quad (3.15)$$

The first equation in the expression — which refers to cases where $c_1 \leq 5000$ — is

$$c_2 = (1.10)(5000 - c_1) + 5500 = 5500 - 1.1c_1 + 5500 = 11000 - 1.1c_1, \quad (3.16)$$

which is precisely what is graphed for $c_1 \leq 5000$ in panel (b) of the graph. The second equation in the expression refers to $c_1 \geq 5000$ and becomes

$$c_2 = (1.05)(5000 - c_1) + 5500 = 5250 - 1.05c_1 + 5500 = 10750 - 1.05c_1. \quad (3.17)$$

If you extended the line to the left of E in panel (b) of Graph 3.6 to the vertical axis, you would indeed find a vertical intercept of 10,750 with the slope of -1.05 leading to a horizontal intercept of approximately 10,238.

3.9 Business Application: Present Value of Winning Lottery Tickets: The introduction to intertemporal budgeting in this chapter can be applied to thinking about the pricing of basic financial assets. The assets we will consider will differ in terms of when they pay income to the owner of the asset. In order to know how much such assets are worth, we have to determine their present value — which is equal to how much current consumption such an asset would allow us to undertake.

A: Suppose you just won the lottery and your lottery ticket is transferable to someone else you designate — i.e. you can sell your ticket. In each case below, the lottery claims that you won \$100,000. Since you can sell your ticket, it is a financial asset, but depending on how exactly the holder of the ticket received the \$100,000, the asset is worth different amounts. Think about what you would be willing to actually sell this asset for by considering how much current consumption value the asset contains — assuming the annual interest rate is 10%.

- (a) The holder of the ticket is given a \$100,000 government bond that “matures” in 10 years. This means that in 10 years the owner of this bond can cash it for \$100,000.

Answer: To know how much this lottery ticket is worth, we have to determine how much the bank would be willing to lend us for current consumption. If we could get the \$100,000 one year from now, we know the bank would lend us up to $\$100,000/1.1 = \$90,909.09$. If we get the \$100,000 two years from now, however, the most the bank would be willing to lend us is $\$100,000/(1.1^2) = \$82,644.63$. And if we can only get to the \$100,000 ten years from now, the most we can get for it now is $(\$100,000)/(1.1^{10}) = \$38,554.33$. Thus, that's the least you would be willing to sell the bond (and thus your ticket) for — and the most anyone else who faces a 10% interest rate should be willing to pay.

- (b) The holder of the ticket will be awarded \$50,000 now and \$50,000 ten years from now.

Answer: The most you can borrow on an amount \$50,000 ten years from now is the amount $\$50,000/(1.1^{10}) = \$19,277.16$. Thus, together with the \$50,000 the lottery awards you now, the most you could consume now is \$69,277.16. Thus, that is the least you would be willing to sell your ticket for.

- (c) The holder of the ticket will receive 10 checks for \$10,000 — one now, and one on the next 9 anniversaries of the day he/she won the lottery.

Answer: For a check n years from now, I can borrow $\$10,000/(1.1^n)$. Calculating this for each of the next 9 years, the checks will be worth \$9,090.91, \$8,264.46, \$7,513.15, \$6,830.13, \$6,209.21, \$5,644.74, \$5,131.58, \$4,665.07 and \$4,240.98. Summing these and adding the value of my current \$10,000 check, the total possible consumption I can undertake is then \$67,590.24 — which is the current value of the ticket.

- (d) How does your answer to part (c) change if the first of 10 checks arrived 1 year from now, with the second check arriving 2 years from now, the third 3 years from now, etc.?

Answer: The value of the first 9 checks would then be the same as the value of the last 9 checks in the previous part. But you would lose the first check from the previous part (which was worth \$10,000) to be replaced with a check 10 years from now, which is worth $\$10,000/(1.1^{10}) = \$3,885.43$. Thus, the ticket would be worth $\$10,000 - \$3,885.43 = \$6,114.57$ less, or \$61,445.67 instead of \$67,590.24.

- (e) The holder of the ticket gets \$100,000 the moment he/she presents the ticket.

Answer: This ticket is, of course, the only one that's worth \$100,000 as claimed by the lottery.

B: More generally, suppose the lottery winnings are paid out in installments of x_1, x_2, \dots, x_{10} , with payment x_i occurring $(i - 1)$ years from now. Suppose the annual interest rate is r .

- (a) Determine a formula for how valuable such a stream of income is in present day consumption — i.e. how much present consumption could you undertake given that the bank is willing to lend you money on future income?

Answer: The present consumption c that could be financed by such a stream of payments is

$$c = x_1 + \frac{x_2}{(1+r)} + \frac{x_3}{(1+r)^2} + \frac{x_4}{(1+r)^3} + \dots + \frac{x_9}{(1+r)^8} + \frac{x_{10}}{(1+r)^9} \quad (3.18)$$

which can also be written as

$$c = \sum_{i=1}^{10} \frac{x_i}{(1+r)^{i-1}}. \quad (3.19)$$

(b) Check to make sure that your formula works for each of the scenarios in part A.

Answer: Plugging in the appropriate values for each part, you should get the same answers as you did in part A.

(c) The scenario described in part A(c) is an example of a \$10,000 payment followed by an annual “annuity” payment. Consider an annuity that promises to pay out \$10,000 every year starting 1 year from now for n years. How much would you be willing to pay for such an annuity?

Answer: The present consumption that could be financed by such an annuity is

$$c = \frac{10000}{(1+r)} + \frac{10000}{(1+r)^2} + \frac{10000}{(1+r)^3} + \dots + \frac{10000}{(1+r)^{n-1}} + \frac{10000}{(1+r)^n}, \quad (3.20)$$

which can also be written as

$$c = \sum_{i=1}^n \frac{10000}{(1+r)^i} = 10000 \sum_{i=1}^n \frac{1}{(1+r)^i}. \quad (3.21)$$

(d) How does your answer change if the annuity starts with its first payment now?

Answer: All that happens is that the expressions in the previous part get an additional \$10,000 added, which would allow us to write the second expression as

$$c = \sum_{i=0}^n \frac{10000}{(1+r)^i} = 10000 \sum_{i=0}^n \frac{1}{(1+r)^i}. \quad (3.22)$$

(e) What if the annuity from (c) is one that never ends? (To give the cleanest possible answer to this, you should recall from your math classes that an infinite series of $1/(1+x) + 1/(1+x)^2 + 1/(1+x)^3 + \dots = 1/x$.) How much would this annuity be worth if the interest rate is 10%?

Answer: Using our equation (3.21) from part (c), we can write this as an infinite series

$$c = 10000 \sum_{i=1}^{\infty} \frac{1}{(1+r)^i} = \frac{10000}{r}. \quad (3.23)$$

If the interest rate is $r = 0.1$, then this expression tells us the annuity would be worth \$100,000.

3.10 Business Application: Picking Savings Accounts: Suppose you just won \$10,000 in the lottery. You decide to put it all in a savings account.

A: Bank A offers you a 10% annual interest rate that compounds annually, while Bank B offers you a 10% annual interest rate compounded every 6 months.

- (a) How much will you have in the bank at the end of the year if you go with Bank A?

Answer: You would have $\$10000(1.1) = \$11,000$.

- (b) How much will you have if you put your money into Bank B?

Answer: A 10% annual interest rate compounded every 6 months means that every 6 months you will get 5% interest on the balance in your account. Leaving it in for 1 year is the same as leaving it in over 2 periods with a period-interest rate of 5% compounded at the end of each period. After 6 months, you will then have \$10,500 in the bank, and after another 6 months, you earn 5% interest on that \$10,500 for a total balance at the end of the year of \$11,025. You can also calculate this simply as $\$10000(1.05^2) = \$11,025$.

- (c) What annual interest rate would Bank A have to offer to make you indifferent between accepting Bank B's and Bank A's offers over the coming year?

Answer: An interest rate of 10.25% compounded annually would result in the same balance one year from now as what you will have at Bank B.

- (d) Would the interest rate you calculated in (c) be sufficient for you to be indifferent between Bank A and Bank B if you planned to keep your money in the savings account for 2 years?

Answer: Yes, it would. We have already determined that, after 1 year, you would have the same balance in either bank — \$11,025. Leaving the money for a second year implies you will have $\$11,025(1.1025) = \$12,155.06$ in Bank A at the end of the second year — which is equal to the amount $\$11,025(1.05^2) = \$12,155.06$ you would have in Bank B.

B: Suppose you place x in a savings account and assume that the account gives an annual interest rate of r compounded n times per year.

- (a) Derive the general formula for how much y you will have accumulated 1 year from now in terms of x , n and the annual interest rate r . Check the answers you derived in (a) and (b) of part A.

Answer: This is

$$y = x \left(1 + \frac{r}{n} \right)^n. \quad (3.24)$$

In A(a), $x = 10000$, $r = 0.1$ and $n=1$, while in A(b), $x = 10000$, $r = 0.1$ and $n=2$. Plugging these in gives the answers above.

- (b) If $x = 10,000$ and $r=0.1$, how much will you have at the end of the year if interest compounds monthly (i.e. $n = 12$)?

Answer: Plugging these into (3.24), we get

$$y = 10000 \left(1 + \frac{0.1}{12} \right)^{12} = 11,047.13. \quad (3.25)$$

- (c) What if interest compounds weekly?

Answer: Then

$$y = 10000 \left(1 + \frac{0.1}{52} \right)^{52} = 11,050.65. \quad (3.26)$$

- (d) If you have to choose between an annual interest rate of 10.5% compounded annually or an annual interest rate of 10% compounded weekly, which would you choose?

Answer: At 10.5% compounded annually, you would have \$11,050 one year from now. Compounded weekly, we just figured out above that you would have \$11,050.65 — just slightly more. So you would presumably choose the latter.

3.11 Business Application: Compound Interest over the Long Run: Uncle Vern has just come into some money — \$100,000 — and is thinking about putting this away into some investment accounts for a while.

A: Vern is a simple guy — so he goes to the bank and asks them what the easiest option for him is. They tell him he could put it into a savings account with a 10% interest rate (compounded annually).

- (a) Vern quickly does some math to see how much money he'll have 1 year from now, 5 years from now, 10 years from now and 25 years from now assuming he never makes withdrawals. He doesn't know much about compounding — so he just guesses that if he leaves the money in for 1 year, he'll have 10% more; if he leaves it in 5 years at 10% per year he'll have 50% more; if he leaves it in for 10 years he'll have 100% more and if he leaves it in for 25 years he'll have 250% more. How much does he expect to have at these different times in the future?

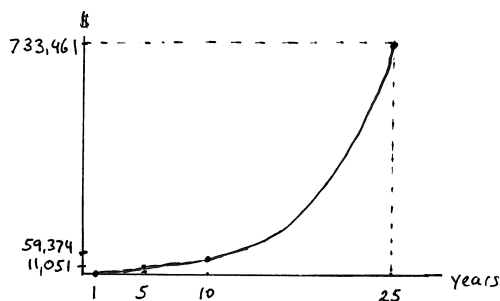
Answer: He expects to have \$110,000 1 year from now, \$150,000 five years from now, \$200,000 ten years from now and \$350,000 twenty-five years from now.

- (b) Taking the compounding of interest into account, how much will he really have?

Answer: Using our usual formula, the actual balance n years from now is $\$100,000(1.1)^n$. This gives \$110,000 one year from now, \$161,051 five years from now, \$259,374.25 ten years from now, and \$1,083,460.59 twenty five years from now.

- (c) On a graph with years on the horizontal axis and dollars on the vertical, illustrate the size of Vern's error for the different time intervals for which he calculated the size of his savings account.

Answer: The size of the error is \$0 one year from now, \$11,051 five years from now, \$59,374.25 ten years from now and \$733,460.59 twenty-five years from now. This is graphed in Graph 3.7.



Graph 3.7: Error from not compounding over time

- (d) True/False: Errors made by not taking the compounding of interest into account expand at an increasing rate over time.

Answer: The statement is clearly true based on the answers above.

B: Suppose that the annual interest rate is r .

- (a) Assuming you will put x into an account now and leave it in for n years, derive the implicit formula Vern used when he did not take into account interest compounding.

Answer: Letting y_n denote the amount he projected will be in the savings account n years from now, he used the formula $y_n = x(1 + nr)$.

- (b) What is the correct formula that includes compounding.

Answer: Using z_n to determine the actual amount in the savings account n years from now, the correct formula is $z_n = x(1 + r)^n$.

- (c) Define a new function that is the difference between these. Then take the first and second derivatives with respect to n and interpret them.

Answer: The new function is $z_n - y_n = x(1+r)^n - x(1+nr)$. First, note that, when $n = 1$, this function reduces to zero — implying the difference between Vern's prediction and reality is zero 1 year from now (just as you determined earlier in the problem). The derivative of this function with respect to n is

$$\frac{\partial(z_n - y_n)}{\partial n} = x(1+r)\ln(1+r) - xr.^1 \quad (3.27)$$

For any $n \geq 1$, this is clearly positive — which means the difference is increasing with time. The second derivative with respect to n is $x(1+r)^n[\ln(1+r)]^2$ which is also positive. Thus the rate at which the difference increases is increasing with time. This is exactly what we illustrated in Graph 3.7.

¹Note that for $y = a^x$, $dy/dx = a^x \ln a$. My thanks to Xiaoyang Zhuang for pointing out an earlier error I made in neglecting to apply these rules of differentiation.

3.12 Business Application: Pricing Government Bonds: A relative sends you a U.S. government savings bond that matures in n years with a face value of \$100. This means that the holder of this bond is entitled to collect \$100 from the government n years from now.

A: Suppose the interest rate is 10%.

- (a) If $n = 1$, how much current consumption could this bond finance and how much do you therefore think you could sell this bond for today?

Answer: You would be able to finance $\$100/(1.1) = \90.91 in current consumption — which is what you could then sell the bond for.

- (b) Does the bond become more or less valuable if the interest rate falls to 5%?

Answer: The value of the bond would be $\$100/(1.05) = \95.24 . Thus, as the interest rate falls, the value of the bond increases. This is generally true — bond prices and interest rates are inversely related.

- (c) Now suppose that $n = 2$. How valuable is the bond if the interest rate is 10%?

Answer: The value of the bond would be $\$100/(1.1^2) = \82.64 .

- (d) What if $n = 10$?

Answer: The value of the bond would then fall to $\$100/(1.1^{10}) = \38.55 .

B: Consider a bond that matures n years from now with face value x when the expected annual interest rate over this period is equal to r .

- (a) Derive the general formula for calculating the current consumption that could be financed with this bond.

Answer: The general formula is $x/(1+r)^n$.

- (b) Use a derivative to show what happens to the value of a bond as x changes.

Answer: Taking the derivative with respect to x , we get $1/(1+r)^n$. Thus, for every additional dollar of face value on the bond, the value of the bond increases by $\$1/(1+r)^n$.

- (c) Show similarly what happens to the value as r changes. Can you come to a general conclusion from this about the relationship between the interest rate and the price of bonds?

Answer: The derivative with respect to r is

$$\frac{-nx}{(1+r)^{n+1}}. \quad (3.28)$$

Since this is negative, we have shown the general proposition that the value of the bond decreases as the interest rate increases.

3.13 Business Application: Buying Houses with Annuities: Annuities are streams of payments that the owner of an annuity receives for some specified period of time. The holder of an annuity can sell it to someone else who then becomes the recipient of the remaining stream of payments that are still owed.

A: Some people who retire and own their own home finance their retirement by selling their house for an annuity. The buyer agrees to pay \$x per year for n years in exchange for becoming the owner of the house after n years.

- (a) Suppose you have your eye on a house down the street owned by someone who recently retired. You approach the owner and offer to pay her \$100,000 each year (starting next year) for 5 years in exchange for getting the house in 5 years. What is the value of the annuity you are offering her assuming the interest rate is 10%?

Answer: The value would be

$$\frac{\$100,000}{1.1} + \frac{\$100,000}{1.1^2} + \frac{\$100,000}{1.1^3} + \frac{\$100,000}{1.1^4} + \frac{\$100,000}{1.1^5} = \$379,078.68. \quad (3.29)$$

- (b) What if the interest rate is 5%?

Answer: Now the value would be

$$\frac{\$100,000}{1.05} + \frac{\$100,000}{1.05^2} + \frac{\$100,000}{1.05^3} + \frac{\$100,000}{1.05^4} + \frac{\$100,000}{1.05^5} = \$432,947.67. \quad (3.30)$$

- (c) The house's estimated current value is \$400,000 (and your real estate agent assures you that homes are appreciating at the same rate as the interest rate.) Should the owner accept your deal if the interest rate is 10%? What if it is 5%?

Answer: Since the house appreciates at the interest rate, we can use its current value and compare it to the current value of the annuity. Given what we calculated above, accepting the annuity is a good deal for the current owner at the 5% interest rate but not at the 10% interest rate.

- (d) True/False: The value of an annuity increases as the interest rate increases.

Answer: This is false, as we just demonstrated above. The value of an annuity increases as the interest rate falls. That should make sense — if I have a given amount to invest, I can invest it where it earns interest, or I can buy an annuity. When the interest rate falls, I will not be able to make as much by investing the money where it makes interest. So that should mean getting a fixed payment through an annuity becomes more valuable.

- (e) Suppose that, after making the second payment on the annuity, you fall in love with someone from a distant place and decide to move there. The house has appreciated in value (from its starting value of \$400,000) by 10% each of the past two years. You no longer want the house and therefore would like to sell your right to the house in 3 years in exchange for having someone else make the last 3 annuity payments. How much will you be able to get paid to transfer this contract to someone else if the annual interest rate is always 10%?

Answer: After two years, the house will be worth $\$400,000(1.1^2) = \$484,000$. The value of the annuity with 3 more payments is

$$\frac{\$100,000}{1.1} + \frac{\$100,000}{1.1^2} + \frac{\$100,000}{1.1^3} = \$248,685.20. \quad (3.31)$$

Thus, you should be able to get $\$484,000 - \$248,685.20 = \$235,314.80$ for selling your contract.

B: In some countries, retirees are able to make contracts similar to those in part A except that they are entitled to annuity payments until they die and the house only transfers to the new owner after the retiree dies.

- (a) Suppose you offer someone whose house is valued at \$400,000 an annual annuity payment (beginning next year) of \$50,000. Suppose the interest rate is 10% and housing appreciates in value at the interest rate. This will turn from a good deal to a bad deal for you when the person lives n number of years. What's n? (This might be easiest to answer if you open a spreadsheet and you program it to calculate the value of annuity payments into the future.)

Answer: If the person lives for another 16 years, the value of the annuity is \$391,185.43. If the person lives an addition year (for a total of 17 years), the value of the annuity becomes \$401,077.67. Thus $n = 17$ because it is a good deal for you as long as the person lives fewer than 17 more years.

- (b) *Recalling that the sum of the infinite series $1/(1+x) + 1/(1+x)^2 + 1/(1+x)^3 + \dots$ is $1/x$, what is the most you would be willing to pay in an annual annuity if you want to be absolutely certain that you are not making a bad deal?*

Answer: If you approximate a long life by infinity, then you would want to pay an annual amount no greater than an amount x that solves the equation $\$400,000 = x/0.1$. Solving this for x gives $x = \$40,000$.

3.14 Business Application: A Trick for Calculating the Value of Annuities: In several of the above exercises, we have indicated that an infinite series $1/(1+r) + 1/(1+r)^2 + 1/(1+r)^3 + \dots$ sums to $1/r$. This can (and has, in some of the B-parts of exercises above) been used to calculate the value of an annuity that pays x per year starting next year and continuing every year eternally as x/r .

A: Knowing the information above, we can use a trick to calculate the value of annuities that do not go on forever. For this example, consider an annuity that pays \$10,000 per year for 10 years beginning next year, and assume $r = 0.1$.

- (a) First, calculate the value of an annuity that begins paying \$10,000 next year and then every year thereafter (without end).

Answer: The value of such an annuity would be

$$\frac{\$10,000}{1.1} + \frac{\$10,000}{1.1^2} + \frac{\$10,000}{1.1^3} + \dots = \frac{\$10,000}{0.1} = \$100,000. \quad (3.32)$$

- (b) Next, suppose you are given such an annuity in 10 years; i.e. suppose you know that the first payment will come 11 years from now. What is the consumption value of such an annuity today?

Answer: We know that such an annuity is worth \$100,000 the year before the payments start; i.e. in 10 years. Anything that is worth \$100,000 ten years from now is worth $\$100,000/(1.1)^{10} = \$38,554.33$ today.

- (c) Now consider this: Think of the 10-year annuity as the difference between an infinitely lasting annuity that starts making payments next year and an infinitely lasting annuity that starts 11 years from now. What is the 10-year annuity worth when you think of it in these terms?

Answer: The infinitely-lasting annuity with payments starting next year is worth \$100,000. The infinitely-lasting annuity with payments starting 11 years from now is worth \$38,554.33. Thus, a 10-year annuity with payments starting next year is worth

$$\$100,000 - \$38,554.33 = \$61,445.67. \quad (3.33)$$

- (d) Calculate the value of the same 10-year annuity without using the trick above. Do you get the same answer?

Answer: Without using the trick, we would have to calculate this as

$$\frac{\$10,000}{1.1} + \frac{\$10,000}{1.1^2} + \frac{\$10,000}{1.1^3} + \dots + \frac{\$10,000}{1.1^{10}}, \quad (3.34)$$

which sums to \$61,445.67. It should make sense that these two methods result in the same outcome. If we had not known the sum of the infinite sequence described at the beginning of the problem, we would have calculated the value of an infinitely-lasting annuity with payments starting next year as

$$\left[\frac{\$10,000}{1.1} + \frac{\$10,000}{1.1^2} + \frac{\$10,000}{1.1^3} + \dots + \frac{\$10,000}{1.1^{10}} \right] + \left[\frac{\$10,000}{1.1^{11}} + \frac{\$10,000}{1.1^{12}} + \dots \right], \quad (3.35)$$

and the value of an infinitely-lasting annuity with payments starting 11 years from now would have been calculated as

$$\left[\frac{\$10,000}{1.1^{11}} + \frac{\$10,000}{1.1^{12}} + \dots \right]. \quad (3.36)$$

Subtracting the equation (3.36) from (3.35) then gives equation (3.34). The trick of viewing a finite annuity as the difference between two infinitely lasting annuities therefore gives us precisely what we know the formula for a finitely lasting annuity must be.

B: Now consider more generally an annuity that pays x every year beginning next year for a period of n years when the interest rate is r . Denote the value of such an annuity as $y(x, n, r)$.

- (a) Derive the general formula for $y(x, n, r)$ by using the trick described in part A.

Answer: The value of an infinitely-lasting annuity that pays x per year (starting one year from now) is x/r . The value of such an infinitely-lasting annuity n years from now is

$$\frac{x/r}{(1+r)^n} = \frac{x}{r(1+r)^n}. \quad (3.37)$$

We can then determine the value of the n -year annuity described in the problem as the difference between two infinitely-lasting annuities; i.e.

$$y(x, n, r) = \frac{x}{r} - \frac{x}{r(1+r)^n} = \frac{x((1+r)^n - 1)}{r(1+r)^n} \quad (3.38)$$

- (b) *Apply the formula to the following example: You are about to retire and have \$2,500,000 in your retirement fund. You can take it all out as a lump sum, or you can choose to take an annuity that will pay you (and your heirs if you pass away) \$ x per year (starting next year) for the next 30 years. What is the least x has to be in order for you to choose the annuity over the lump sum payment assuming an interest rate of 6%.*

Answer: Substituting $n = 30$ and $r = 0.06$ into the formula for the value of the annuity, we get

$$\frac{x((1.06)^{30} - 1)}{0.06(1.06)^{30}} = 13.76483x. \quad (3.39)$$

The only way you will accept the annuity is if its value is at least \$2,500,000; i.e.

$$13.76483x \geq 2,500,000. \quad (3.40)$$

Solving for x , we get \$181,622.30. Thus, the annuity has to pay at least this much per year.

- (c) *Apply the formula to another example: You can think of banks as accepting annuities when they give you a mortgage. Suppose you determine you would be able to pay at most \$10,000 per year in mortgage payments. Assuming an interest rate of 10%, what is the most the bank will lend you on a 30-year mortgage (where the mortgage payments are made annually beginning 1 year from now)?*

Answer: The bank would be willing to lend you

$$\frac{\$10000((1 + 0.10)^{30} - 1)}{0.1(1 + 0.1)^{30}} = \$94,269.14. \quad (3.41)$$

- (d) *How does your answer change when the interest rate is 5%?*

Answer: When the interest rate is $r = 0.05$, the formula becomes

$$\frac{\$10000((1 + 0.05)^{30} - 1)}{0.05(1 + 0.05)^{30}} = \$153,724.51. \quad (3.42)$$

- (e) *Can this explain how people in the late 1990's and early 2000's were able to finance increased current consumption as interest rates fell?*

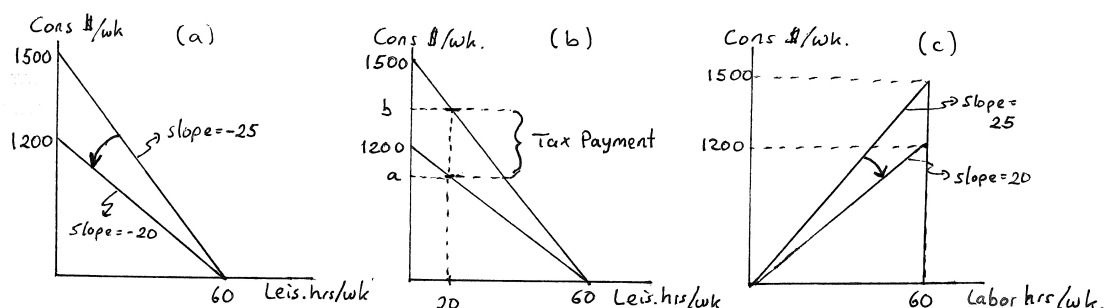
Answer: Suppose the bank originally lent you \$94,269 at 10% interest with you making \$10,000 in mortgage payments per year. Then the interest rate falls to 5% — and you can now re-finance and borrow \$153,725 and still only make \$10,000 in annual mortgage payments. Thus, you have additional money for consumption by re-financing. This was a major source of increased consumption in the U.S. in the late 1990's and early 2000's as people re-financed their homes given low interest rates — and consumed the additional amount they were able to borrow. (Another big part of the story was that home values were increasing — also allowing consumers to borrow more on their home equity.)

3.15 Policy Application: Wage Taxes and Budget Constraints. Suppose you have 60 hours of leisure that you could devote to work per week, and suppose that you can earn an hourly wage of \$25.

A: Suppose the government imposes a 20% tax on all wage income.

- (a) Illustrate your weekly budget constraint before and after the tax on a graph with weekly leisure hours on the horizontal and weekly consumption (measured in dollars) on the vertical axis. Carefully label all intercepts and slopes.

Answer: The two budget constraints are illustrated in panel (a) of Graph 3.8. The after-tax wage is $w = 20$, 20% less than the before tax wage $w = 25$.



Graph 3.8: Wage Taxes and Tax Payments

- (b) Suppose you decide to work 40 hours per week after the tax is imposed. How much wage tax do you pay per week? Can you illustrate this as a vertical distance in your graph? (Hint: Follow a method similar to that developed in end-of-chapter exercise 2.15.)

Answer: This is illustrated in panel (b) of the graph. When you work 40 hours a week, you consume 20 hours of leisure. On the after-tax budget, that leaves you with a in consumption. On the before-tax budget, it leaves you with b in consumption. The vertical difference $(b - a)$ must therefore be the total tax payment you made under the tax. Note that $a = 800$ and $b = 1000$ — with the vertical difference therefore equal to $(b - a) = 200$. This makes sense — when the worker is working for 40 hours a week, he is earning \$1,000 before taxes and thus pays \$200 in taxes at a 20% tax rate.

- (c) Suppose that instead of leisure hours on the horizontal axis, you put labor hours on this axis. Illustrate your budget constraints that have the same information as the ones you drew in (a). Assume again that the leisure endowment is 60 per week.

Answer: This is illustrated in panel (c) of the graph.

B: Suppose the government imposes a tax rate t (expressed as a rate between 0 and 1) on all wage income.

- (a) Write down the mathematical equations for the budget constraints and describe how they relate to the constraints you drew in A(a).

Answer: For every hour of labor, a worker makes w but pays tw in taxes. Thus, his after-tax wage is $(1 - t)w$. He will be able to consume as much as he earns, and how much he earns depends on how much leisure he does not consume. Letting ℓ be leisure hours consumed, this implies that $c = (1 - t)w(60 - \ell)$. For the case where $w = 25$ and $t = 0.2$, this becomes $c = 0.8(25)(60 - \ell) = 1200 - 20\ell$ which is the after-tax equation we graphed in panel (a). The before-tax equation has $t = 0$, with $c = 25(60 - \ell) = 1500 - 25\ell$.

- (b) Use your equation to verify your answer to part A(b).

Answer: Using the after-tax equation $c = 1200 - 20\ell$, we can plug in $\ell = 20$ which is when you choose to work 40 hours per week. This gives $c = 1200 - 20(20) = 800$ which is the consumption level denoted a in panel (b) of the graph. Using the before-tax equation $c = 1500 - 25\ell$,

we get $c = 1500 - 25(20) = 1000$ which is the consumption level denoted b in panel (b) of the graph. The difference between the two consumption levels is \$200 — which is the tax payment per week. This is intuitively correct — if you work for 40 hours at a wage of \$25, you earn \$1,000 per week, and if you pay taxes of 20% on that, you will pay \$200 in taxes.

- (c) *Write down the mathematical equations for the budget constraints you derived in B(a) but now make consumption a function of labor, not leisure hours. Relate this to your graph in A(c).*

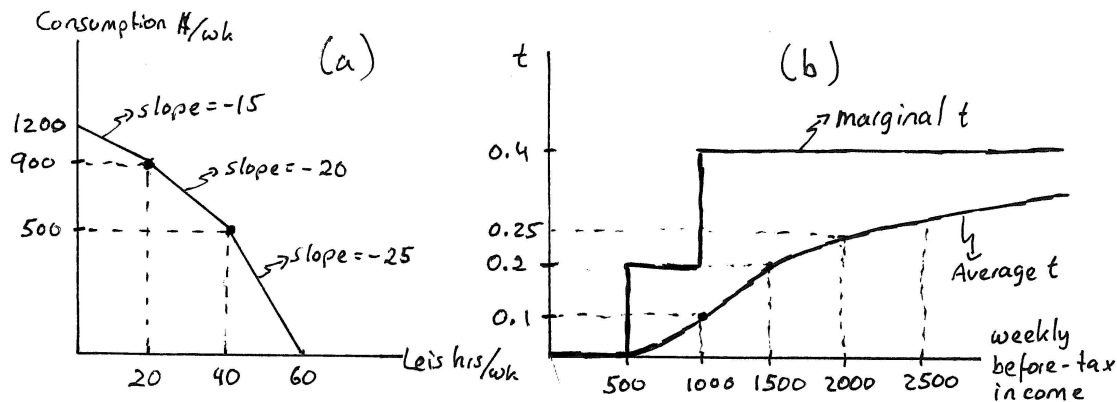
Answer: Let labor hours be denoted l . Then, with a tax t and wage w , your consumption c is simply the portion of your pay check that you get to keep — which is $(1 - t)wl$. Thus, $c = (1 - t)wl$. When $w = 25$ and $t = 0.2$, this becomes $c = 0.8(25)l = 20l$ which is the after tax budget constraint graphed in panel (c) of Graph 3.8.

3.16 Policy Application: Proportional versus Progressive Wage Taxes: The tax analyzed in exercise 3.15 is a proportional wage tax. The U.S. federal income tax, however, is progressive. This means that the average tax rate one pays increases the more wage income is earned.

A: For instance, suppose the government exempts the first \$500 of weekly earnings from taxation, then taxes the next \$500 at 20% and any earnings beyond that at 40%. Suppose that you again have 60 hours of leisure per week and can earn \$25 per hour.

(a) Graph your weekly budget constraint illustrating the tradeoffs between leisure and consumption.

Answer: Panel (a) of Graph 3.9 illustrates this budget constraint. It takes 20 hours to earn \$500 of weekly earnings when the wage is \$25 per hour. Thus, the kink points produced by the progressive rate structure occur at 20 hours of work (i.e. 40 hours of leisure) and 40 hours of work (i.e. 20 hours of leisure.)



Graph 3.9: Progressive Wage Tax

(b) The marginal tax rate is defined as the tax rate you pay for the next dollar you earn, while the average tax rate is defined as your total tax payment divided by your before-tax income. What is your average and marginal tax rate if you choose to work 20 hours per week?

Answer: If you work for 20 hours per week (i.e. you consume 40 hours of leisure), your marginal tax rate is 0.2 because you will be taxed 20% on the next dollar earned. Your average tax rate, however, is zero since you have paid no tax up to this point.

(c) How does your answer change if you work 30 hours? What if you work 40 hours?

Answer: If you work 30 hours (i.e. you consume 30 hours of leisure), your marginal tax rate is still 0.2. Your average tax rate, however, is your total tax payments divided by your before tax income; i.e. $50/750 = 0.0667$ or 6.67%. If you work 40 hours (i.e. you consume 20 hours of leisure), your marginal tax rate jumps to 0.4 because you are making \$1000 per week which means that the next dollar will be taxed at 40%. At this point you will have paid 20% on \$500 income — or \$100 in taxes. Thus, your average tax rate is $100/1000 = 0.10$ or 10%.

(d) On a graph with before-tax weekly income on the horizontal axis and tax rates on the vertical, illustrate how average and marginal tax rates change as income goes up. Will the average tax rate ever reach the top marginal tax rate of 0.4?

Answer: This is graphed in panel (b) of Graph 3.9. As income increases, the average tax rate will approach the marginal tax rate, but because the first \$500 is exempt from taxes and the second \$500 is taxed at half the top marginal rate, the average will never quite reach the marginal rate.

- (e) *Some have proposed that the U.S. should switch to a “flat tax” — a tax with one single marginal tax rate. Proponents of this tax reform typically also want some initial portion of income exempt from taxation. The flat tax therefore imposes two different marginal tax rates: a tax rate of zero for income up to some amount x per year, and a single rate t applied to any income earned above x per year. Is such a tax progressive?*

Answer: Because of the initial exemption, the average tax rate will always increase with income (just as in the previous case) even though there is a single marginal tax rate that kicks in above the exemption. Thus, the tax is progressive.

B: *Suppose more generally that the government does not tax income below x per week; that it taxes income at t for anything above x and below $2x$, and it taxes additional income (beyond $2x$) at $2t$. Let I denote income per week.*

- (a) *Derive the average tax rate as a function of income and denote that function $a(I, t, x)$ — where I represents weekly income.*

Answer: The function is given by

$$a(I, t, x) = \begin{cases} 0 & \text{if } I \leq x \text{ and} \\ t(I - x)/I & \text{if } x < I \leq 2x \text{ and} \\ (tx + 2t(I - 2x))/I & \text{if } I > 2x. \end{cases} \quad (3.43)$$

- (b) *Derive the marginal tax rate function $m(I, t, x)$.*

Answer: The marginal tax rate function is

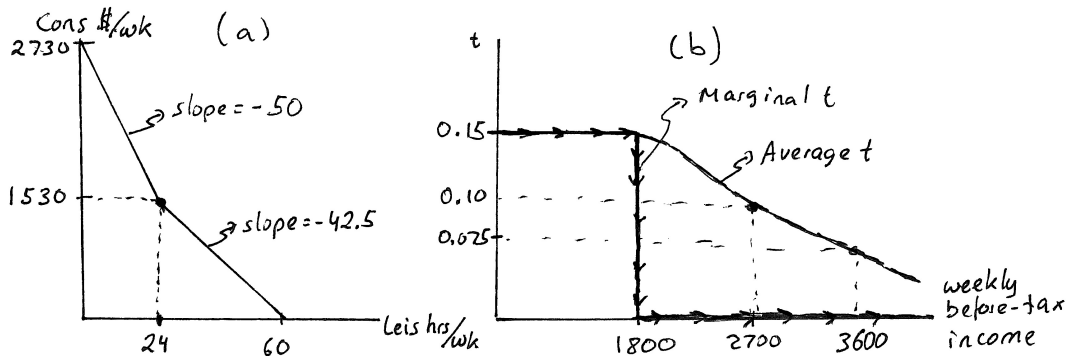
$$m(I, t, x) = \begin{cases} 0 & \text{if } I < x \text{ and} \\ t & \text{if } x \leq I < 2x \text{ and} \\ 2t & \text{if } I \geq 2x. \end{cases} \quad (3.44)$$

3.17 Policy Application: Social Security (or Payroll) Taxes: Social Security is funded through a payroll tax that is separate from the federal income tax. It works in a way similar to the following example: For the first \$1,800 in weekly earnings, the government charges a 15% wage tax but then charges no payroll tax for all earnings above \$1,800 per week.

A: Suppose that a worker has 60 hours of leisure time per week and can earn \$50 per hour.

- (a) Draw this worker's budget constraint with weekly leisure hours on the horizontal axis and weekly consumption (in dollars) on the vertical.

Answer: Panel (a) of Graph 3.10 traces out this budget constraint. The kink point happens at 24 hours of leisure — or 36 hours of labor. At that point, the worker earns \$1800 before taxes and pays $0.15(\$1800) = \270 in taxes, leaving him with \$1,530 in consumption. For any lower levels of leisure (more work), the worker incurs no additional tax, causing his budget constraint to get steeper.



Graph 3.10: Regressive Payroll Tax

- (b) Using the definitions given in exercise 3.16, what is the marginal and average tax rate for this worker assuming he works 30 hours per week? What if he works 40 hours per week? What if he works 50 hours per week?

Answer: If he works 30 hours, his marginal and average tax rates are both 0.15 or 15%. If he works 40 or 50 hours, his marginal tax rate is zero. His before tax income at 40 hours is \$2,000 and at 50 hours it is \$2,500. In both cases, he pays \$270 in weekly payroll taxes. Thus, his average tax rate at 40 hours of work is $270/2000 = 0.135$ or 13.5%. His average tax rate at 50 hours of work is $270/2500 = 0.108$ or 10.8%.

- (c) A wage tax is called regressive if the average tax rate falls as earnings increase. On a graph with weekly before-tax income on the horizontal axis and tax rates on the vertical, illustrate the marginal and average tax rates as income increases. Is this tax regressive?

Answer: This is graphed in panel (b) of Graph 3.10. Taxes with declining average tax rates are regressive — so yes, this tax is regressive.

- (d) True or False: Budget constraints illustrating the tradeoffs between leisure and consumption will have no kinks if a wage tax is proportional. However, if the tax system is designed with different tax brackets for different incomes, budget constraints will have kinks that point inward when a wage tax is regressive and kinks that point outward when a wage tax is progressive.

Answer: This is true as illustrated in this exercise and exercise 3.16.

B: Consider the more general case of a tax that imposes a rate t on income immediately but then falls to zero for income larger than x .

(a) Derive the average tax rate function $a(I, t, x)$ (where I represents weekly income).

Answer: The function is

$$a(I, t, x) = \begin{cases} t & \text{if } I \leq x \text{ and} \\ (tx)/I & \text{if } x < I. \end{cases} \quad (3.45)$$

(b) Derive the marginal tax rate function $m(I, t, x)$.

Answer: The marginal tax rate function is

$$m(I, t, x) = \begin{cases} t & \text{if } I < x \text{ and} \\ 0 & \text{if } x \geq I. \end{cases} \quad (3.46)$$

(c) Does the average tax rate reach the marginal tax rate for high enough income?

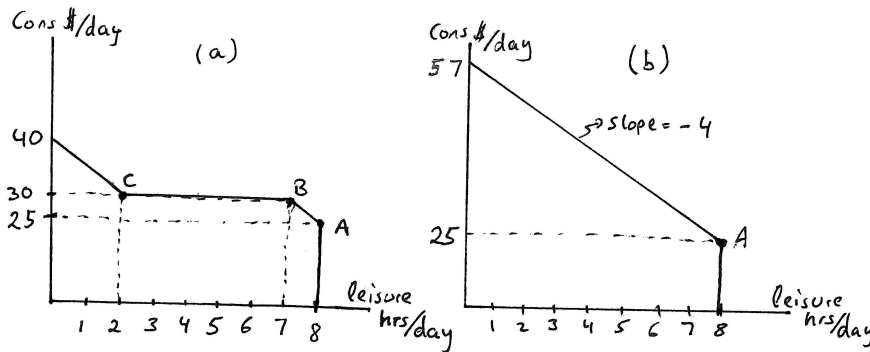
Answer: No, it merely converges to 0 as income gets large but never reaches it because everyone always continues to pay tx regardless of how high income gets.

3.18 Policy Application: AFDC versus a Negative Income Tax: Until the late 1990's, one of the primary federal welfare programs was AFDC — or Aid to Families with Dependent Children. The program was structured roughly similarly to the following example: Suppose you can work any number of hours you choose at \$5 per hour and you have no income other than that which you earn by working. If you have zero overall income, the government pays you a welfare payment of \$25 per day. You can furthermore receive your full welfare benefits so long as you make no more than a total income of \$5 per day. For every dollar you earn beyond \$5, the government reduces your welfare benefits by exactly a dollar until your welfare benefits go to zero.

A: Suppose you have up to 8 hours of leisure per day that you can dedicate to work.

(a) Draw your budget constraint between daily leisure and daily consumption (measured in dollars).

Answer: Panel (a) of Graph 3.11 plots this budget constraint. It begins at bundle A with 8 hours of leisure and \$25 in welfare benefits. Since welfare benefits do not get reduced for the first \$5 earned, one less hour of leisure translates into \$30 rather than \$25 of consumption — bundle B in the graph. But from then on, until all \$25 in welfare benefits are exhausted, decreased leisure (i.e. increased work) does not translate into additional consumption. At bundle C, all welfare payments are gone and the worker again benefits from reducing his leisure.



Graph 3.11: AFDC versus a Negative Income Tax

(b) If you define marginal tax rates in this example as the fraction of additional dollars earned in the labor market that a worker does not get to keep, what is the marginal tax rate faced by this worker when he is working 1 hour per day? What if he is working 5 hours per day? What if he is working 6 hours a day?

Answer: The marginal tax rate faced by a worker who is working 1 hour per day is 1.00, or 100%. This is because welfare benefits are reduced dollar for dollar as additional income is earned in the labor market. The same is true for a worker who works 5 hours per day. Once the worker is working 6 hours per day, the marginal tax rate falls from 100% to zero.

(c) Without knowing anything about tastes, how many hours are you likely to work under these tradeoffs?

Answer: It seems likely that a worker will work no more than 1 hour given that he faces a 100% tax rate for the next five hours of work.

(d) The late Milton Friedman was critical of the incentives in the AFDC program and proposed a different mechanism for supporting the poor. He suggested a program, known as the negative income tax, that works something like this: Everyone is guaranteed \$25 per day that he/she receives regardless of how much he/she works. Every dollar from working, starting with the

first one earned, is then taxed at $t = 0.2$. Illustrate our worker's budget constraint assuming AFDC is replaced with such a negative income tax.

Answer: Panel (b) of Graph 3.11 graphs the negative income tax budget constraint. It begins at the same point A as the AFDC budget and then increases at a rate of \$4 per hour worked because the 20% tax begins immediately (thus causing the after-tax wage to be \$4 per hour.)

- (e) Which of these systems will almost certainly cost the government more for this worker — the AFDC system or the negative income tax? Which does the worker most likely prefer? Explain.

Answer: The AFDC system almost certainly costs more. This is because it is likely that the worker will work at most 1 hour under the AFDC system — and will collect the entire \$25 in welfare. Under the negative income tax, on the other hand, it is likely that the worker will work more — and will thus pay some taxes that will reduce the net-payment to him under the negative income tax below \$25. Since almost the entire AFDC budget lies within the negative income tax budget, it is likely that the worker would prefer the negative income tax. (Only a small corner of the AFDC budget around point B would stick out of the negative income tax budget if the two were laid one on top of the other. It seems quite likely that the guaranteed income could be raised sufficiently for the entire AFDC budget to be contained in the negative income tax budget and still the government pays out less under the negative income tax than AFDC.)

- (f) What part of your negative income tax graph would be different for a worker who earns \$10 per hour?

Answer: The constraint would still begin at A but would go up at a steeper rate — at \$8 per hour worked (given that the before tax wage of \$10 falls to \$8.)

- (g) Do marginal tax rates for an individual differ under the negative income tax depending on how much leisure he/she consumes? Do they differ across individuals?

Answer: No, the marginal tax rate under the negative income tax is always the same — for any individual regardless of how much she works, and across individuals regardless of how much they make.

B: Consider a more general version of the negative income tax, one that provides a guaranteed income y and then reduces this by some fraction t for every dollar earned — resulting eventually in individuals with sufficiently high income paying taxes.

- (a) Derive a general expression for the budget constraint under a negative income tax, a constraint relating daily consumption c (in dollars) to daily leisure hours ℓ assuming that at most 8 hours of leisure are available.

Answer: $c = y + (1 - t)w(8 - \ell)$

- (b) Derive an expression for how much the government will spend (or receive) for a given individual depending on how much leisure she takes.

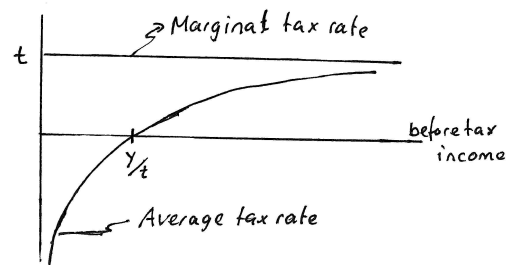
Answer: The government gives y as guaranteed income but then collects $tw(8 - \ell)$ in taxes from this worker. Thus, the overall government payment (or receipt) is $y - tw(8 - \ell)$.

- (c) Derive expressions for marginal and average tax rates as a function of daily income I , the guaranteed income level y and the tax rate t . (Hint: Average tax rates can be negative.)

Answer: As we already determined above, the marginal tax rate is constant and equal to t . The average tax rate is the total in net-tax payments divided by income. An individual pays tI in taxes but also collects y — which implies that the total net-payment is $tI - y$. Thus, the average tax rate is $(tI - y)/I$ which is negative for low levels of I .

- (d) On a graph with daily before-tax income on the horizontal axis and tax rates on the vertical, illustrate how marginal and average tax rates change as income rises.

Answer: Graph 3.12 depicts these, with the average tax rate given by $(tI - y)/I$ as derived above. When $tI = y$, income has risen sufficiently for tax payments to exactly offset the guaranteed income payment. Solved for I , this occurs at $I = y/t$ — which is where the average tax function crosses the intercept because the average tax at that point is zero. The average tax function approaches but never converges to the marginal tax rate t because, no matter how high income gets, everyone receives the guaranteed income y .



Graph 3.12: Average and Marginal Tax Rates under the Negative Income Tax

(e) *Is the negative income tax progressive?*

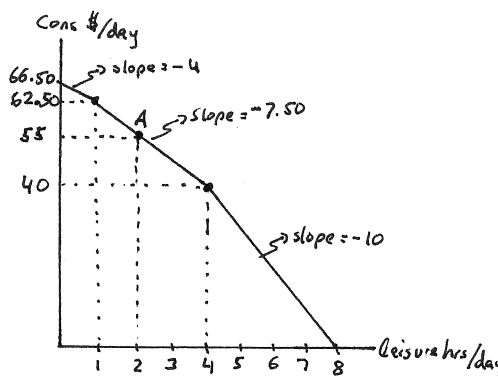
Answer: A tax is progressive if the average tax rate increases with income. This is the case here — so yes, the negative income tax is progressive.

3.19 Policy Application: The Earned Income Tax Credit. During the Clinton Administration, the EITC — or Earned Income Tax Credit, was expanded considerably. The program provides a wage subsidy to low income families through the tax code in a way similar to this example: Suppose, as in the previous exercise, that you can earn \$5 per hour. Under the EITC, the government supplements your first \$20 of daily earnings by 100% and the next \$15 in daily earnings by 50%. For any daily income above \$35, the government imposes a 20% tax.

A: Suppose you have at most 8 hours of leisure time per day.

- (a) Illustrate your budget constraint (with daily leisure on the horizontal and daily consumption on the vertical axis) under this EITC.

Answer: The budget constraint is graphed in Graph 3.13. For the first 4 hours of labor, the take-home wage is \$10 per hour because of the 100% subsidy. For the next 3 hours of labor, the take-home wage is \$7.50 because of the 50% subsidy. Finally, for any work beyond 7 hours, the take home wage is \$4 because of the 20% tax.



Graph 3.13: EITC Budget Constraint

- (b) Suppose the government ends up paying a total of \$25 per day to a particular worker under this program and collects no tax revenue. Identify the point on the budget constraint this worker has chosen. How much is he working per day?

Answer: The worker would work for 6 hours. At a wage of \$5, this would mean making \$30 per day. But, for the first \$20, the government adds \$20, and for the next \$10 hours, the government adds \$5 — for a total EITC supplement of \$25. Thus, the worker will have \$55 in income for other consumption. This gives us A in the graph — leisure of 2 hours per day (because of 6 hours of work) and consumption of \$55 per day.

- (c) Return to your graph of the same worker's budget constraint under the AFDC program in exercise 3.18. Suppose that the government paid a total of \$25 in daily AFDC benefits to this worker. How much is he working?

Answer: The worker is working at most 1 hour.

- (d) Discuss how the difference in trade-offs implicit in the EITC and AFDC programs could cause the same individual to make radically different choices in the labor market.

Answer: Despite the government spending the same on the worker under AFDC and EITC, the worker might choose to not work much under AFDC and a lot under EITC. This is because of the implicit large tax rate imposed on the worker under AFDC but not under EITC.

B: More generally, consider an EITC program in which the first x dollars of income are subsidized at a rate $2s$; the next x dollars are subsidized at a rate s ; and any earnings above $2x$ are taxed at a rate t .

(a) Derive the marginal tax rate function $m(I, x, s, t)$ where I stands for labor market income.

Answer: This function is

$$m(I, x, s, t) = \begin{cases} -2s & \text{if } I < x \text{ and} \\ -s & \text{if } x \leq I < 2x \text{ and} \\ t & \text{if } I \geq 2x. \end{cases} \quad (3.47)$$

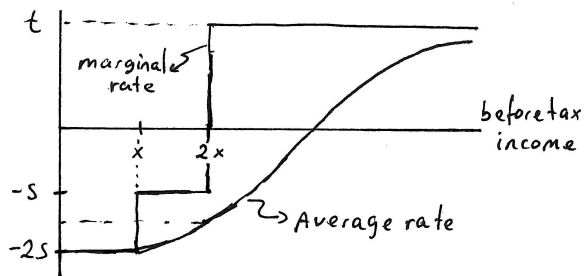
(b) Derive the average tax rate function $a(I, x, s, t)$ where I again stands for labor market income.

Answer: This is

$$a(I, x, s, t) = \begin{cases} -2s & \text{if } I \leq x \text{ and} \\ -[2sx + s(I - x)]/I & \text{if } x < I \leq 2x \text{ and} \\ [-3sx + t(I - 2x)]/I & \text{if } I > 2x. \end{cases} \quad (3.48)$$

(c) Graph the average and marginal tax functions on a graph with before-tax income on the horizontal axis and tax rates on the vertical. Is the EITC progressive?

Answer: This is graphed in Graph 3.14. Since average tax rates rise as income rises, the EITC is progressive.



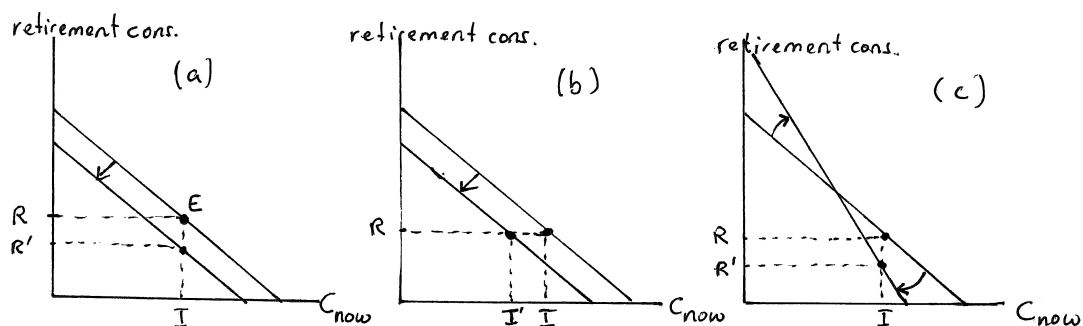
Graph 3.14: Average and Marginal Tax Rates under the EITC

3.20 Policy Application: Three Proposals to Deal with the Social Security Shortfall: It is widely recognized that the social security systems in many western democracies will face substantial shortfalls between anticipated revenues and promised benefits over the coming decades.

A: Various ideas have emerged on how we should prepare for this upcoming shortfall.

- (a) In order to analyze the impact of different proposals, begin with a graph that has “consumption now” on the horizontal and “retirement consumption” on the vertical axes. For simplicity, suppose we can ignore periods between now and retirement. Consider a worker and his choice set over these two “goods”. This worker earns some current income I , and he is currently promised a retirement income R from the government. Illustrate how this establishes an “endowment point” in your graph. Then, assuming an interest rate r over the period between now and retirement, draw this worker's choice set.

Answer: Panel (a) of Graph 3.15 graphs this budget constraint as the one through the endowment point labeled E . The slope is $-(1+r)$.



Graph 3.15: 3 Social Security Reforms

- (b) Some have proposed that we need to cut expected retirement benefits for younger workers — i.e. we need to cut R to $R' < R$. Illustrate the impact this has on our worker's choice set.

Answer: This change is also graphed in panel (a) of the graph. The policy shifts the expected retirement income but does not alter r and thus does not alter the opportunity cost of consuming now versus consuming later. Thus, the slope stays the same and the movement in the endowment point simply results in a parallel inward shift of the budget constraint.

- (c) Others have argued that we should instead raise social security taxes — i.e. reduce I to $I' < I$ — in order to prepare for the upcoming shortfall. Illustrate how this would impact our worker's budget constraint.

Answer: This is illustrated in panel (b). This time, the current income I is shifted but the expected social security income R remains unchanged. Again, nothing has changed the interest rate and thus the opportunity cost of consuming now versus later remains unchanged. We again get an inward parallel shift of the budget constraint as the government policy adjusts the endowment point.

- (d) Assuming that r is not impacted differently by these two policies, could you argue that they are essentially the same policy?

Answer: Both result in a parallel inward shift of budgets. So, as long as the size of the reduction in R on the one hand and of I on the other is comparable, the two policies are identical in their impact on choice sets.

- (e) Yet others have argued that we should lower future retirement benefits R but at the same time subsidize private savings — i.e. increase r — through policies like expanding tax deferred savings accounts. Illustrate the impact of lowering R and raising r .

Answer: This is illustrated in panel (c) of Graph 3.15 where the increase in r causes the slope of the budget to become steeper while the decrease in R causes the endowment point to shift down.

- (f) Which of these policies is the only one that has a chance (though by no means a guarantee) of making some individuals better off?

Answer: Only the last one might make some individuals better off — because only the last policy results in a new budget constraint that might contain some points that were not contained in the original budget constraint.

B: Define I , R and r as above.

- (a) Write down the mathematical description of the current intertemporal budget for our worker — in terms of I , R and r . Let c_1 denote current consumption and let c_2 denote retirement consumption.

Answer: Retirement consumption will depend on how much we save ($I - c_1$) plus the level of social security benefits R that we receive. The budget constraint is therefore $c_2 = (1 + r)(I - c_1) + R = (1 + r)I + R - (1 + r)c_1$.

- (b) In your equation, show which parts correspond to the vertical intercept and slope in your graphs from part A.

Answer: The intercept term is $[(1 + r)I + R]$ and the slope term is $-(1 + r)$.

- (c) Relate your equation to the changes that you identified in the graph from each of the policies.

Answer: When only R is changed (as in panel (a) of Graph 3.15), only the intercept term is affected since R appears there but not in the slope term. The same is true when I is changed (as in panel (b) of Graph 3.15). Finally, when both R and r are changed, both the intercept and slope terms are affected. Whether the new intercept term is higher than the old one (as in the graph) or lower depends on the relative magnitude of the change in R and r .